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Overestimation of Original Gas in Place in Water-Drive Gas Reservoirs Due to a Misleading Linear p/z plot

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ABSTRACT

A straight-line plot of p/z vs. G_p *(Cumulative Gas Production) is widely used to estimate the Original Gas in Place. It is known as the p/z plot technique. The linearity of that plot has been historically known to be a unique feature of a volumetric (closed) reservoir. In this paper, we show that a uniqueness problem may exist when using the p/z plot. In other words, if the reservoir is in contact with an aquifer, a straight-line may exist on that plot causing a major overestimation of Original Gas in Place. This uniqueness problem is proved to be due solely to the unsteady-state nature of aquifers*

A simulation study was performed to determine the conditions for such a misleading straight line. Several examples demonstrate that it is possible to construct a synthetic data set for a water-drive gas reservoir such that a misleading straight-line plot is obtained. This misleading straight-line is shown to be due to certain rate schedules. The conventional material balance equation is coupled with an aquifer mathematical model to obtain this schedule. In this paper, an actual field case is presented as an example of this possible overestimation of Original Gas in Place due to a misleading linear p/z plot.

INTRODUCTION

The material balance equation is an expression of the law of the conservation of mass, which is commonly used in reservoir engineering. For reservoirs with no water influx and no water encroachment and if we neglect formation and water compressibility's, it will have the following form

$$GB_g = (G - G_p)B_g \quad (1)$$

Which can be written also as

$$\frac{p}{z} = \frac{p_i}{z_i}(1 - \frac{G_p}{G}) \quad (2)$$

If we include all the forces that we previously neglected, then we will have the following equation

$$GB_{gi} = (G - G_p)B_g + W_e - W_p + GB_{gi}(1 - R_M) \quad (3)$$

Which can be written as

$$\frac{p}{z} = \frac{p_i}{z_i} \frac{\left(1 - \frac{Gp}{G}\right)}{\left(R_M - \frac{W_e}{GB_{gi}}\right)} \quad (4)$$

If we neglect the influence of the formation and water compressibility then Ramagost factor R_M will be equal to 1, and therefore equation (4) will be reduced to become as follows

$$\frac{p}{z} = \frac{p_i}{z_i} \frac{\left(1 - \frac{Gp}{G}\right)}{\left(1 - \frac{W_e}{GB_{gi}}\right)} \quad (5)$$

We can re-write equation (2) to be simplified in the following form

$$\frac{p}{z} = \frac{p_i}{z_i} - CG_p \quad (6)$$

where the constant ,

$$C = \frac{p_i}{z_i G} \quad (7)$$

Now, equation 6 is of interest to us because it states that, there is a linear relationship between *p/z* and the cumulative volume of produced gas G_p as shown by the solid line in Fig. 1. If the trend of this straight -line is extrapolated to *p/z* = 0, then we can obtain an estimate of the Original Gas in Place, *G*, since when *p/z* = 0

$$1 - \frac{G_p}{G} = 0, G_p = G$$

It is important at this point to repeat the statement by Chierici[1] concerning equation 2 where he says: " The linear relationship is a necessary but not a sufficient condition for the reservoir to be of the closed type". This study revolves around this statement to explain what we call here, the uniqueness problem in estimating Original Gas in Place.

The usually expected behavior of the performance plots for water-drive gas reservoirs is the curved behavior shown by the black dots in Fig. 1. This is simply because we are now using equation 5 instead of equation 2 where the term W_e is introduced that accounts for the cumulative water influx that encroached into the reservoir at a specific time.

In Fig. 2, we see that the data points for the water-drive gas reservoir can lay on a straight-line (the dashed line), which can be falsely interpreted in the field to be of a volumetric reservoir. This line when extrapolated will lead to obtain an overestimated Original Gas in Place (*G'*).

FIELD EXAMPLE

We will now consider a *p/z* plot obtained by using data from a gas field. This field is located in the Gulf of Mexico and we will refer to it here as Field A. The plot seemed to give an approximately linear trend to any engineer. The wrong assumption that the reservoir is of the volumetric type, is what led to an overestimation of Original Gas in Place.

Field A was a relatively new field when the data was analyzed. The linear trend of the *p/z* plot shown in Fig. 3 by the solid line ruled out any possibility that there may be a natural water drive. Accordingly, the engineer in the operating company gave an estimate of 270 Bcf for Original Gas in Place. This estimate turned out, as we will see to be very optimistic. The estimate obtained using the aquifer analysis methods and the approach suggested by Vega[2] gave an estimate of 110 Bcf for

Original Gas in Place as shown by the dotted line in Fig. 3.

In Fig. 4, we see a zoomed view of the data points plotted on Fig. 3. Again, we emphasize that any engineer could see the *p/z* plot sufficient enough to consider this being a straight-line and any scatter might be just regarded to be due to measurement errors. At the current time, we have updated information about this reservoir. After a couple of more years of production, one of its three wells of the field has watered out and the current estimate for the Original Gas in Place is 170 Bcf. It is our belief that the misinterpretation of the driving mechanism of the reservoir that led to this overestimated Original Gas in Place, was due to the performance of the rate schedule shown in Fig. 5. The performance shows a trend of increase in production rate, which agrees with results obtained from the analysis results of our study as will be shown later.

MODELING OF AQUIFER INFLUX EFFECT IN THE MATERIAL BALANCE EQUATION

The term W_e is a function of pressure and time and it is calculated usually by assuming elementary shapes for aquifer analytical models (linear or radial aquifers) and also assuming several properties of aquifers. In this study, we assume radial and linear shapes for the aquifers in our synthetic gas reservoir / aquifer cases. The Van Everdingen and Hurst[3] solutions for a transient aquifer were used to account for the water influx effect by using the superposition principle.

The views of many authors[4-10] that have been reported in the literature showed few field and synthetic cases that give an apparent *p/z* plot where $r^2 \cong 1$. They did their work using what we call here a forward model. Conversely, in this study, we show results from what we call an inverse model. Here we assume an exact straight-line *p/z* plot with a false overestimated Original Gas in Place (G'). This is simply because we are interested in the theory behind the existence of the straight-line *p/z* plot in water-drive gas reservoirs.

In order to obtain that certain behavior of a linear performance of the *p/z* vs. G_p that has a slope of $(p_i/z_i)/G'$, we simulated cases using the following two models that were programmed in visual basic for applications

1. Gas reservoir / aquifer forward model

The forward model is used to obtain the plot of *p/z* vs. G_p for a particular assumed production rate schedule that we may enter manually into the program. We keep on manipulating the parameters to be entered in the program to try to obtain an apparent linear performance of *p/z* vs. G_p and we mean by apparent that $r^2 \cong 1$ (i.e. appears linear to the naked eye). Different production rate schedules, can yield the same G' for the same gas reservoir/aquifer system.

2. Gas reservoir / aquifer inverse model

The inverse model assumes a particular exact linear performance ($r^2 = 1$) of the *p/z*-plot that has a slope of $(p_i/z_i)/G'$. We do this by simply assuming a certain false *Original Gas in Place* (G'). We obtain from this model a unique production rate schedule for a particular gas reservoir/aquifer system. This rate schedule consists of a series of reported average constant rates over a specified time period (Δt_p). In this work, we used the period of time (Δt_p) of average constant rate to be one month, except for initial average production rate, which was assumed to be for six months.

RATE DEPENDANCE OF PERFORMANCE PLOTS FOR WATER-DRIVE GAS RESERVOIRS

In this section, we will illustrate an example of a case for a gas reservoir / linear aquifer system where we produce at several constant values of production rate. The properties of this system is shown in Table 1 where we assume that we have a linear aquifer which has an absolute permeability value of 5 md. We observe in Fig. 6, the different performances on the *p/z* plot at different constant production rates for a water-drive gas reservoir. If we remove the effect of the aquifer, i.e. we have a volumetric reservoir, the *p/z* plot will always show the same performance line regardless of the value of the production rate. The fact that water-drive gas reservoirs are rate dependent is due solely to the fact that these reservoirs are directly communicated with aquifers that have an unsteady-state nature, i.e. aquifer influx rate is a function of time. From the mathematical modeling point of view, this fact causes the material balance equation for a water-drive gas reservoir to have time as a variable. This time variable didn't exist in the material balance equation for a volumetric reservoir.

We are interested now to show that theoretically at very early time, the *p/z* plot performance for a water-drive reservoir case lies on the same line of the *p/z* plot for the volumetric reservoir, and thus extrapolates to the correct value of Original Gas in Place. This is an important fact that was also proved when trying to program the inverse model as a straight-line forced between (p_i/z_i) and our choice of G'. We will first start by taking a bigger picture of the area of interest in Fig. 6. The bigger picture can be observed on a magnified scale in Fig. 7 where we notice that especially the lines of the *p/z* plot performance for high rates (producing faster), appear as if they were initially on the line of the volumetric reservoir.

In Appendix A, we show a simple derivation for this early time curvature behavior, that proofs that as time approaches zero (i.e. very early time), the general slope m of the *p/z* plot for a water-drive gas reservoir is reduced to become that of the volumetric reservoir.

RATE SCHEDULES THAT CAUSE AN EXACT MISLEADING STRAIGHT LINE

El-Ahmady[10] showed different examples of water-drive gas reservoirs that may show an apparent (non-exact) linear performance on the *p/z* plot using the forward model (where we assume the rate schedule). These plots look practically linear to the naked eye (similar to Field Case A), and the reservoir may be dangerously confused with one of the volumetric depletion type.

Here we show results from the inverse model, where we demonstrate that theoretically, it is possible when producing with a certain production schedule to obtain a perfect (exact) straight-line on the *p/z* plot. The overestimated Original Gas in Place (G') depends on the value we set for our initial rate. The strength of the aquifer (represented here by the permeability) plays an important role in the shape of our rate schedule behavior. There is a variation in the manner of the production performance depending on the strength of the aquifer and on the initial rate of production. The initial production rate in the first 6 months and the permeability of the aquifer are the two criteria that we manipulated in our base case to demonstrate different results. The initial production rate is an output from the program from the inverse model but we can guess it by setting a G' which we expect to give this initial rate. This guessing method is done using the forward model.

We observe in Fig. 8 the comparison of the different aquifer strengths for the base case of the initial rate of 10 MMscf/D. We notice that as the aquifer strength decreases, we reach the peak rate at a faster time and we have a longer period of production recovery.

We observe in Fig. 9 the comparison of the different initial rates for the base case of the permeability of aquifer, $k_{aq.}$ = 50 md. We notice that as we increase the initial production rate, we reach the peak rate at a faster time.

DISCUSSION

Hundreds of examples can be shown for the apparent (non-exact) misleading linear *p/z* plot. El-Ahmady[10] showed that different rate schedules can exhibit this apparent linearity that when extrapolated may give the same overestimated Original Gas in Place(G'). However, to get an exact misleading *p/z* plot, for a particular gas reservoir/aquifer system, there is only a unique rate schedule for a particular G' as shown in Figs. 8 and 9.

In Fig. 10, we show a schematic of the early time curvature behavior, before point M, where we begin to see the performance of the misleading *p/z*-plot. This period of time, before point M can not be observed when plotting field data, since we don't have pressure measurements at such early time. If several accurate measurement readings for pressure and production data, were available at very early time, then there would have been no uniqueness problem, since we could have observed the curvature behavior, and thus knew that we have a case of a water-drive gas reservoir. This is the reason that we assumed that the first measurement we had was after six months at point M, where the misleading linear performance, begins after that point. This is why we have an average constant rate for the first six months. The inverse model can not be programmed otherwise, because there must be the curvature behavior at very early time for any reservoir in contact with an aquifer,

It is interesting to note that most of the work in the literature done to model the behavior of water-drive gas reservoirs used the constant pressure solution and had large time steps. The minimum time step was that reported by Agarwal[5] which was 6 months, while Bruns et al.[6] used a two year time step.

In our work here, we used the constant rate solution in both the forward and the inverse models, we found that the results are very sensitive to the time step size. This appeared very clearly in cases where the cases where we used the forward model where we assumed a fluctuated (and not a constant) production rate schedule. In this study, we used time steps of 0.5 day for the first 6 months period (where the production is assumed constant) and 1 day for the rest of the production history. However, the programs reported the monthly average rates by calculating for the arithmetic average for the production for all time steps during that month.

To further increase accuracy, we used an averaging method to calculate the water-influx rate e_w. The water-influx rate is represented when plotted versus time as a linear spline function rather than a stair step function. It is our belief that this is a more accurate way to account the aquifer influx effect. In addition to that, it seems closer to the nature of the physics of the aquifer.

We excluded the effects of water and formation compressibility to simplify the analysis of the results but they can be easily included in our models.

CONCLUSIONS

1. An apparent linear performance of a *p/z* plot can exist in practice, especially at the early time of recovery from a field. This has been demonstrated by a field example.
2. An overestimated value of the Original Gas in Place is obtained from the misleading performance of the linear *p/z* plot for a water-drive gas reservoir.
3. At very early times, the *p/z* plot extrapolates to the correct Original Gas in Place. However, this may not be practical because of data errors and lack of pressure measurements at very early time.
4. In this study, for a simulated case of a gas reservoir/aquifer system, it is shown that data points beginning at 6 months, can give an exact misleading straight line on the *p/z* plot, that is caused by a unique production rate schedule.

ACKNOWLEDGEMENT

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NOMENCLATURE

A_c = cross sectional area to flow, L^2, ft^2
B_{gi} = gas FVF at initial pressure, dimensionless, rcf/scf
B_w = water FVF, dimensionless, RB/STB
C = constant
c_t = total compressibility, Lt^2/m, $psia^{-1}$
d = *derivative*
e_w = water influx rate, L^3/t, bbl/day
f = radial enroachment factor, fraction
h = net formation thickness, L, ft
k = permeability, L^2, md
G_p = cumulative gas produced, L^3, scf.
G = Original Gas in Place, L^3, scf
G' = False Original Gas in Place, L^3, scf
m = *slope,* m/L^4t^2, psia/scf
N_p = cumulative oil production, L^3, bbl
p = reservoir pressure, m/Lt^2, psia
PV = Pore Volume
r = statistics regression factor, fraction
r_o = reservoir radius, ft
R_m = Ramagost factor
Δt = time step, t, days
Δt_p = time period of constant production rate, t, days
S_{gr} = residual gas saturation, fraction
S_w = water saturation, fraction

t = time, t, days
T = reservoir temperature, T, ^{0}R
W_e = cumulative water influx, L^3, STB
W_p = cumulative water production, L^3, bbl
z = gas deviation factor
ϕ = porosity, fraction
γ_g = gas specific gravity, fraction
μ_w = water viscosity, m/Lt, cp

Subscripts

aq. = aquifer
i = initial conditions

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APPENDIX A

The coming simple derivation shows the early time curvature behavior, that is, as time approaches zero (i.e. very early time), the general slope *m* of the *p/z* plot for a water-drive gas reservoir is reduced to become that of the volumetric reservoir.

$$\text{Let } m = \frac{d(p/z)}{dG_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(A-1)$$

And,

$$\text{Let } q_g = \frac{dG_p}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(A-2)$$

And,

$$\text{Let } e_w = \frac{dW_e}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(A-3)$$

From the material balance equation for a gas reservoir in contact with an aquifer, we know that

$$\left(\frac{p}{z}\right) = \frac{p_i}{z_i}\frac{\left(1-\frac{G_P}{G}\right)}{\left(R_m-\frac{(W_e-W_p)B_w}{GB_{gi}}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(A-4)$$

Re-arranging the above equation, and assuming R_m = 1, B_w =1, W_p = 0 just for simplicity in writing equations, we deduce that

$$\left(\frac{p}{z}\right)\left(1-\frac{We}{GBgi}\right) = \frac{p_i}{z_i}\left(1-\frac{Gp}{G}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(A-5)$$

Differentiate equation (A-5) by *dt*

$$\frac{d(p/z)}{dt} - \frac{1}{GB_{g_i}}$$

$$\left(\frac{p}{z} \times \frac{dW_e}{dt} + W_e \times \frac{d(p/z)}{dt}\right) = -\frac{p_i}{z_iG}\frac{dGp}{dt} \text{...........}(\text{A} - 6)$$

If we re-arrange equation (A-6), we will get

$$\frac{d(p/z)}{dG_p} \times \frac{dG_p}{dt} - \frac{1}{GB_{g_i}}$$

$$\left(\frac{p}{z} \times \frac{dW_e}{dt} + W_e \times \frac{d(p/z)}{dG_p} \times \frac{dG_p}{dt}\right) = -\frac{p_i}{z_iG}\frac{dGp}{dt} (\text{A} - 7)$$

Substitute equation (A-1) and equation (A-2) into equation (A - 8)

$$mq_g - \frac{1}{GB_{g_i}}\left(\frac{p}{z}\frac{dW_e}{dt} + W_emq_g\right) = -\frac{p_i}{z_iG}q_g \text{ ....(A - 8)}$$

Multiply equation (A-8) by *G*

$$mq_gG - \frac{1}{B_{g_i}}\left(\frac{p}{z}e_w + W_emq_g\right) = -\frac{p_i}{z_i}q_g \text{ ....(A - 9)}$$

Re-arranging equation (A-9)

$$mq_gG - \frac{(p/z)}{B_{g_i}}e_w - \frac{W_emq_g}{B_{g_i}} = -\frac{p_i}{z_i}q_g \text{ ....(A-10)}$$

and we already know that

$$\frac{(p/z)}{B_{g_i}} = \frac{(p_i/z_i)}{B_g} \text{ ....(A-11)}$$

Therefore, from equations (A-10) and (A-11)

$$mq_gG - \frac{p_i}{z_i}\frac{e_w}{B_g} - \frac{W_emq_g}{B_{g_i}} = -\frac{p_i}{z_i}q_g \text{ ....(A-12)}$$

Re-arranging equation (A-12)

$$mq_g\left(G - \frac{W_e}{B_{g_i}}\right) = \frac{p_i}{z_i}\left(\frac{e_w}{B_g} - q_g\right) \text{ ....(A-13)}$$

Re-arranging equation (A-13)

$$mq_g\left(G - \frac{We}{Bgi}\right) = \frac{p_i}{z_i}q_g\left(\frac{e_w}{q_gB_g} - 1\right) \text{ ....(A-14)}$$

Re-arranging equation (A-14)

$$m = \frac{p_i}{z_i}\frac{(e_w/q_gB_g - 1)}{\left(G - \frac{W_e}{B_{g_i}}\right)} \text{ ....(A-15)}$$

At *t* approaches 0, e_w and W_e both approach 0 initially. Therefore, the equation reduces to become

$$m = \frac{-p_i/z_i}{G} \text{ ....(A-16)}$$

which is the equation of the straight-line for a volumetric reservoir.

Table 1-Synthetic Data for a gas reservoir in contact with a linear aquifer

G	60 Bcf	k_{aq}	5 md (absolute, aquifer)
p_i	5000 psia	c_t	3 x 10^{-6} / psi (total, aquifer)
h	100 ft (reservoir/aquifer)	A_c	1936 ft (reservoir/aquifer)
ϕ	0.24 (reservoir/aquifer)	T	200°F
S_{wc}	0.2 *PV*	B_w	1.0 RB/STB
S_{gr}	0.2 *PV*	W_p	0
γ_g	0.65 (air = 1)	N_p	0
μ_w	0.40 cp		

Table 2 – Synthetic Data for a gas reservoir in contact with a radial aquifer

G	60 Bcf	k_{aq}	20, 50 and 100 md (absolute, aquifer)
p_i	5000 psia	c_t	3 x 10^{-6} / psi (total, aquifer)
h	100 ft (reservoir/aquifer)	f	1 (360° radial encroachment)
ϕ	0.24 (reservoir/aquifer)	r_o	1936 ft (reservoir radius)
S_{wc}	0.2 PV	T	200°F
S_{gr}	0.2 PV	B_w	1.0 RB/STB
γ_g	0.65 (air = 1)	W_p	0
μ_w	0.40 cp	N_p	0

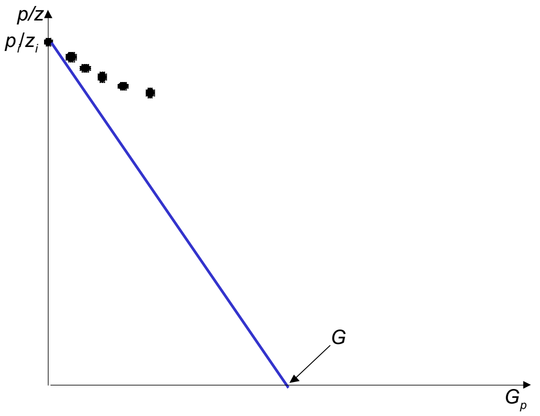

Fig. 1 Schematic plot of the traditionally expected behavior of the *p/z* plot

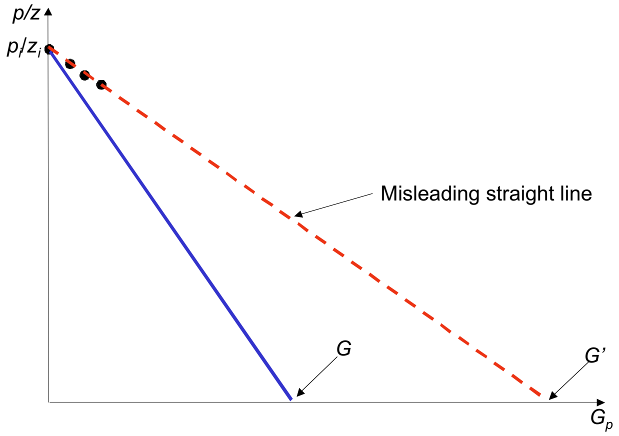

Fig. 2 Schematic plot of the misleading *p/z* plot

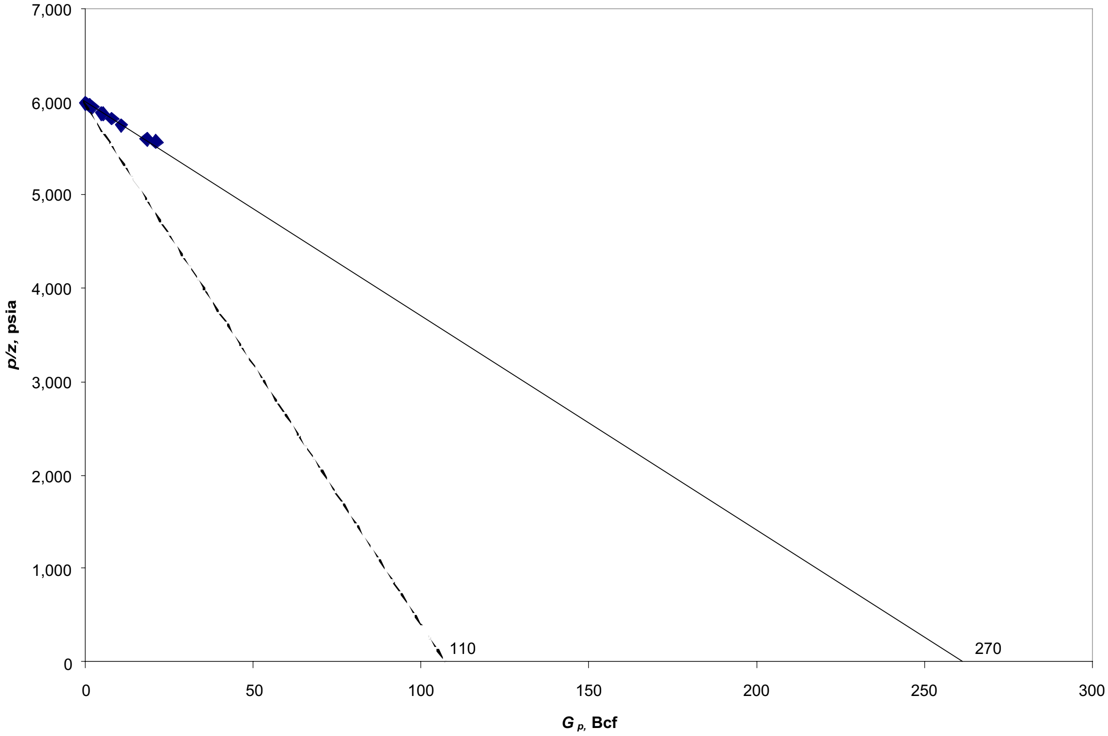

Fig. 3 - Apparent linear *p/z* plot for Field A

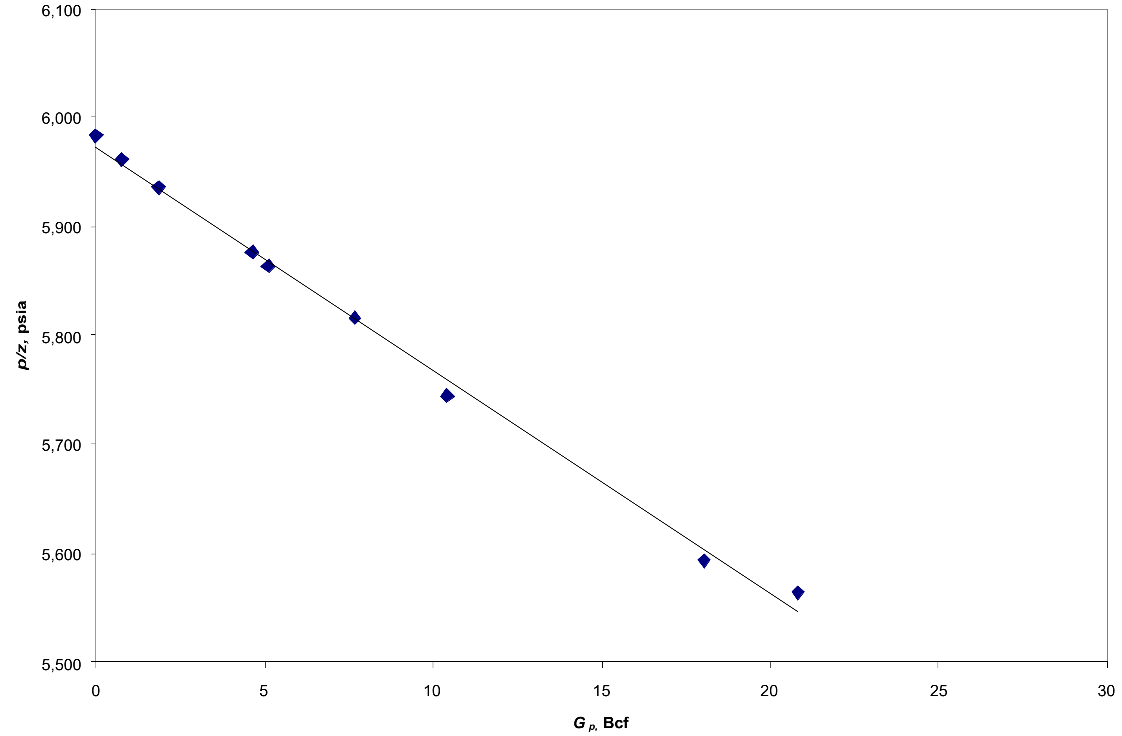

Fig. 4 - Zoomed view of the data points for *p/z* plot for Field A

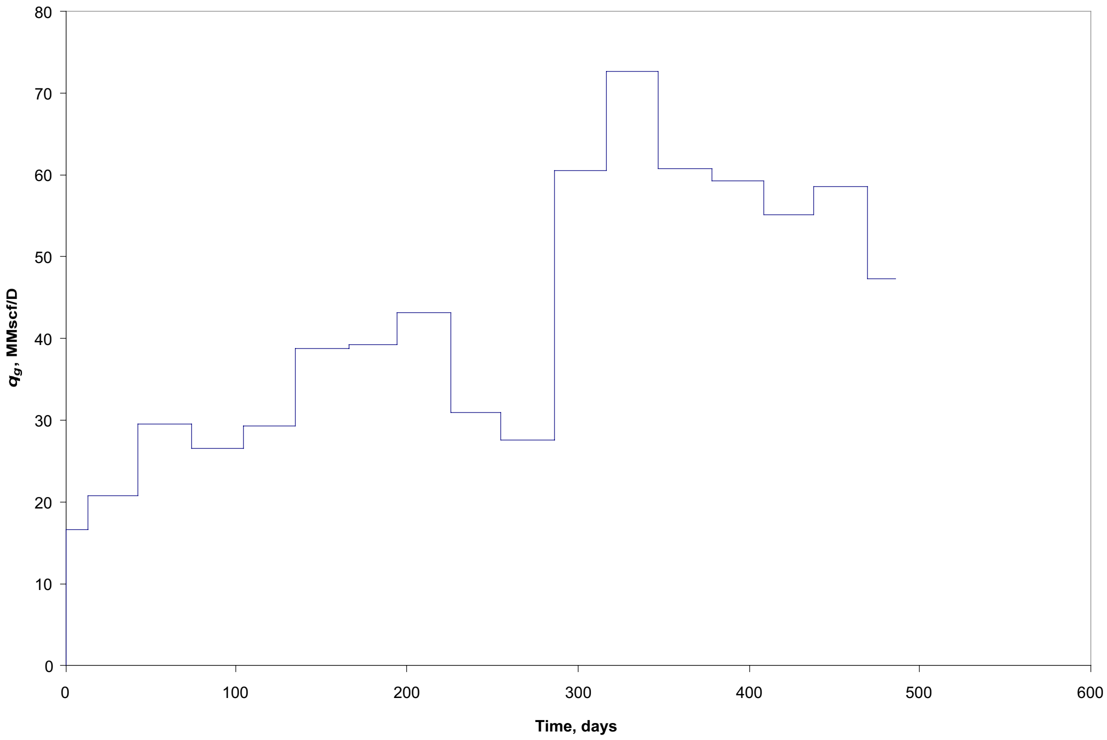

Fig. 5 - Production rate profile for Field A

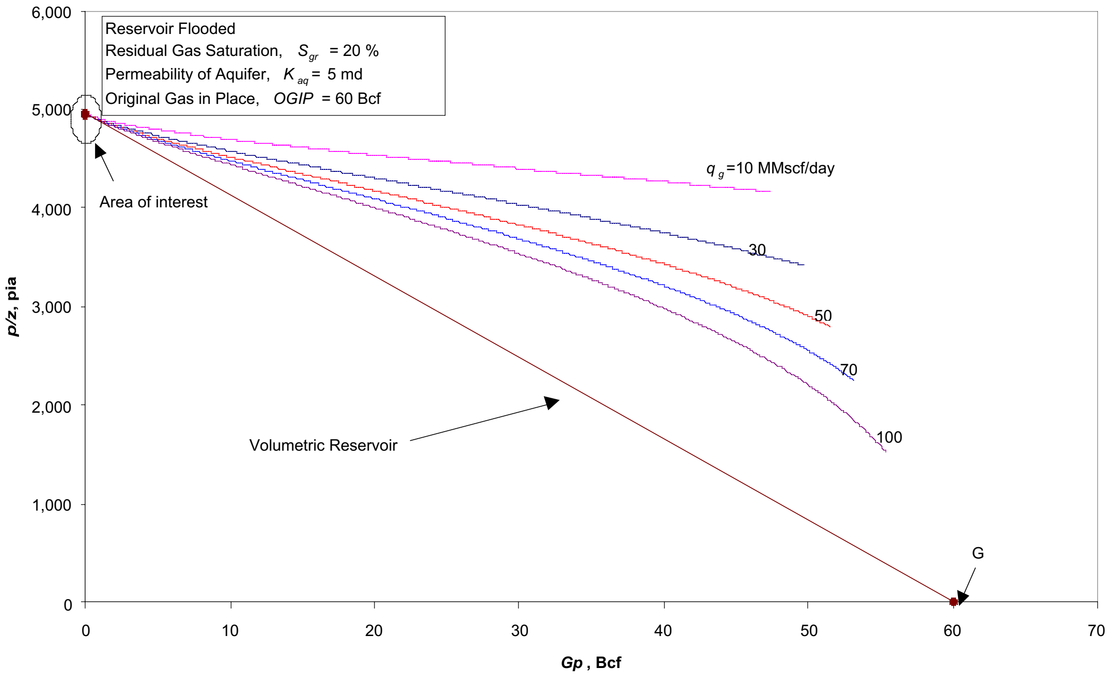

Fig. 6 - Performance of *p/z* plot at different constant production rates

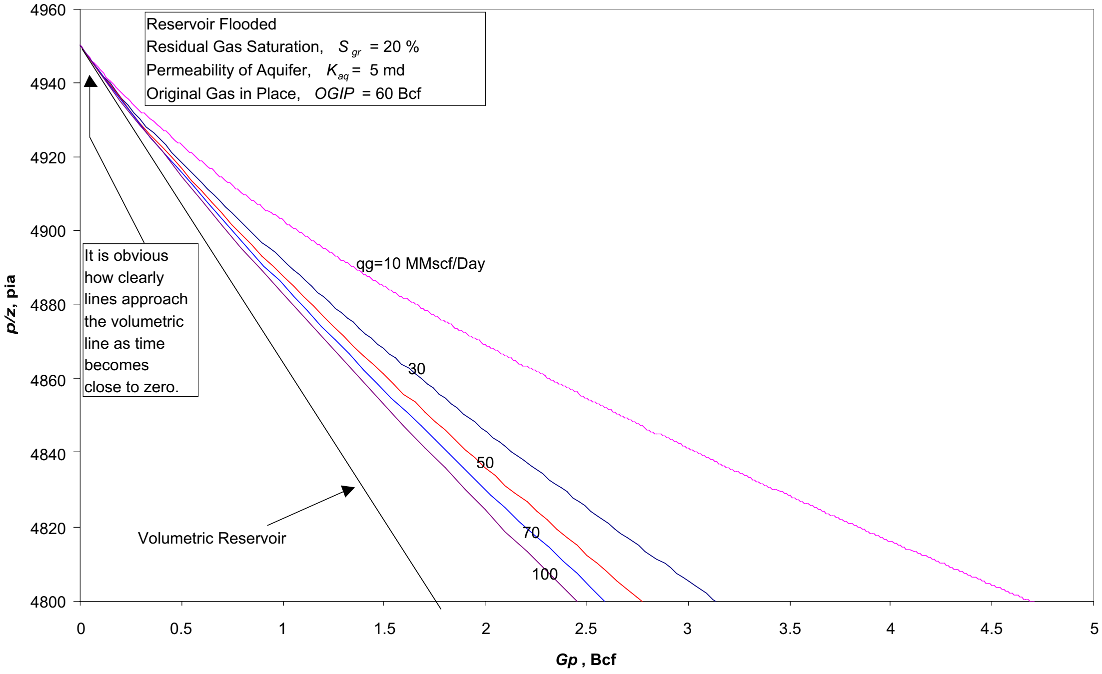

Fig. 7 - A magnified plot of area of interest in Fig. 6

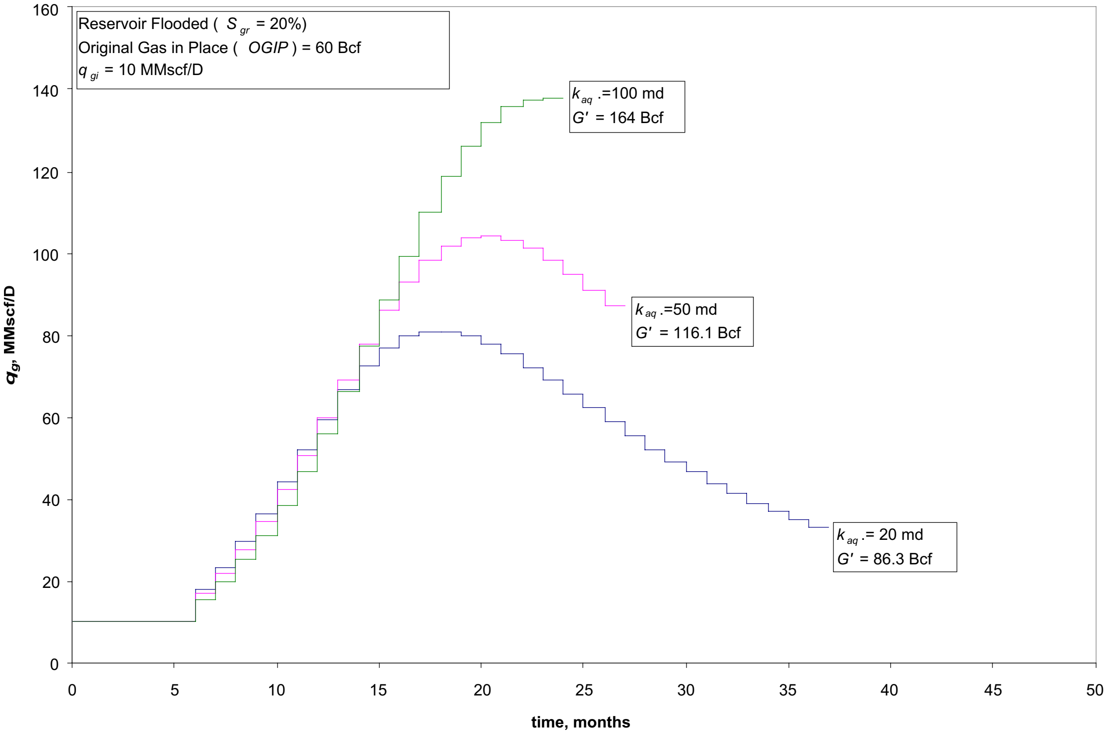

Fig. 8- Rate Schedules for different $K_{aq.}$ That yield different G' values

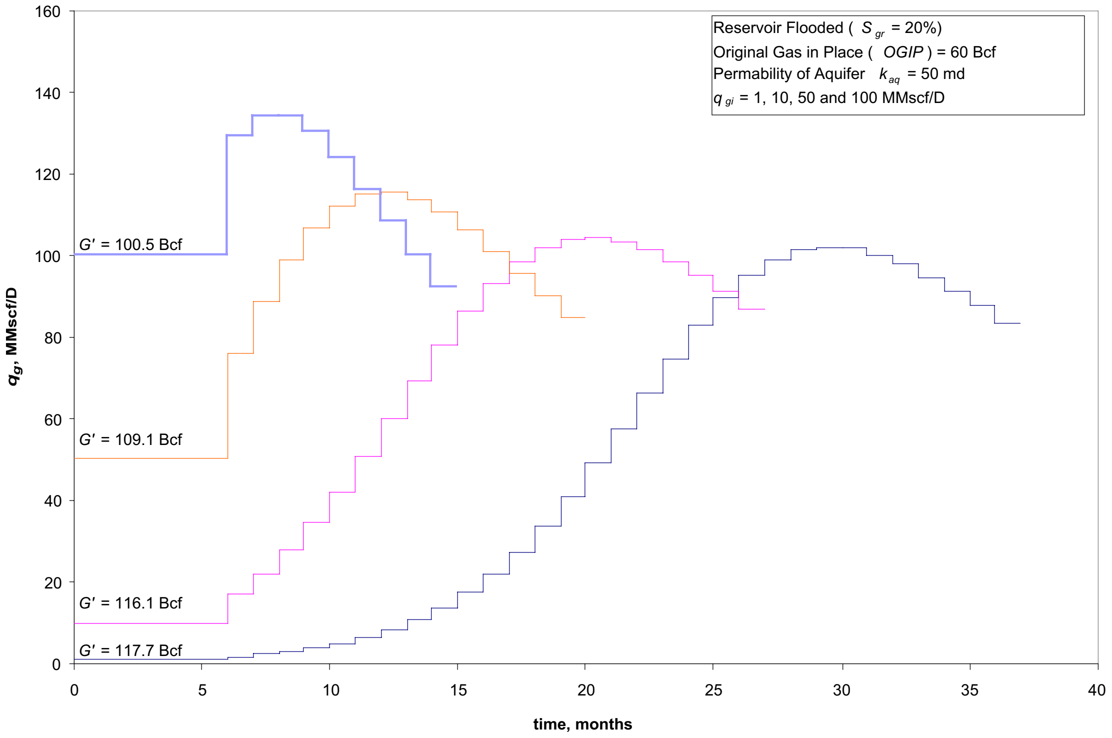

Fig. 9- Rate Schedules for different q_{gi} that yield different G' values

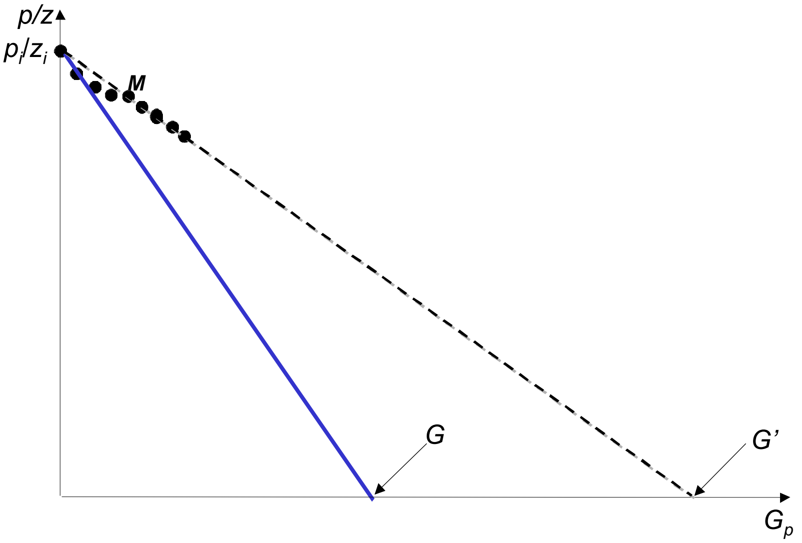

Fig. 10- Schematic plot showing the early curvature, followed by the misleading linear behavior